Using Videos of Teaching to Study Teacher Knowledge

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Purpose of the video study

As part of a larger study on the knowledge teachers need to teach algebra, our research team decided to explore video of algebra teaching to seek evidence of teacher knowledge and how it is used in practice. After initially planning to go to classrooms and capture new video, we decided instead to seek existing video from other projects that we could use.

The problem

We began this research with two primary questions:

1. How can we conceptualize the knowledge teachers use when teaching algebra?
2. How can we use video data from classrooms to study teacher knowledge?

The first question is a directed at the substance of the larger project, development of a framework for studying teacher knowledge. We used video both to develop and to test the emerging framework.

The second question is a methodological one. We aimed to create a stronger theoretical argument, and an articulated method, for using video to study teacher knowledge.

Sources of data about teacher knowledge

Proxies for knowledge (such as number of mathematics courses or quality of undergraduate institution) have yielded only weak correlations with student outcomes.
(Begle, 1972, 1979; Monk, 1994). Because the argument that teachers’ subject matter knowledge has an impact on student learning is logical and believable, one suspects that these proxies do not represent what teachers need to know to be effective mathematics teachers. If there is knowledge for teaching algebra that makes a difference in student learning, we do not yet know what it is or how to measure it.

A logical source for understanding teacher knowledge is practice itself: what do teachers do in their classrooms and how do their actions reflect or represent their knowledge? Other research has used such an approach (Ball & Bass, 2000) with theoretical and practical success. In their work on teacher knowledge, Ball and colleagues used video of teaching, mining it for possible evidence of teacher knowledge. They developed conjectures about teacher knowledge, and eventually used this deep background in practice, and the theoretical constructs they generated, to create measures of teacher knowledge that are proving to be useful in understanding effective teaching in elementary school mathematics (Hill et al., 2004).

Taking a similar approach, this project used videos of algebra teaching that could be mined for conjectures about teacher knowledge. We approached this problem with questions not only about knowledge itself, but also about the nature of video: what are the characteristics of video that might make it useful for studying teacher knowledge?

**Method: How to “see” knowledge**

Knowledge is not visible: it cannot be observed directly. How can we possibly “see” knowledge when we watch video? Our approach has been to use video as a provocation for discussion about teacher knowledge. We identify incidents that clearly call on knowledge of algebra, and develop conjectures about what we take to be the
knowledge a teacher might use or possess, based on what we see her do or say. We do not use the video to analyze a particular teacher’s knowledge, but as a catalyst for generating claims about the knowledge that the particular incident might entail, given specific contextual constraints.

For example, take the case of Cindy, described in more detail below (Seago et al., 2004). We have two different versions of the same lesson, taught by Cindy and separated by a year; pre and post interviews with Cindy for each of the two lessons; transcripts and lesson graphs showing details of the flow of the lessons. Yet, even with relatively extensive data, it is difficult to sort out the following:

1. What is the role or situation of this lesson in the curriculum? What is the curriculum?
2. What is the purpose of this lesson within the larger unit of instruction?
3. What commitments and beliefs does Cindy bring to this teaching, with respect to the nature of mathematical knowledge, how students learn mathematics, how teachers should teach mathematics, and what is important for students to know and be able to do?

Other aspects of the teaching are also unknown, because they are not part of the video or the accompanying records: How did Cindy plan this lesson? What materials were available to her before the lesson? What happened in class in the days before the lesson? What happened in the days after the lesson? All these questions bear on how we can understand Cindy’s knowledge, both what she actually knew and what it might have been useful for her to know.
In response to the inherently incomplete nature of video, we developed a schema for investigating teacher knowledge, illustrated in Figure 1. We begin with what we know about the video itself, providing answers to as many of the contextual questions as possible. We develop this scenario to the extent allowed by the data (e.g., what we can learn from the video and accompanying materials about the context). Next we develop one or more alternate scenarios that vary the assumed or known context of the video. For example, if we do not know how the lesson fits into the curriculum, we create one or more alternatives that might yield different views of teacher knowledge. Then, we view the video with these different scenarios in mind, looking for incidents that are particularly revealing. From these alternate scenarios, we develop alternate “packages” of knowledge that might be called on. All of this is done in light of the emerging framework for understanding knowledge for algebra teaching. Finally our conjectures about knowledge packages are used to reflect on and revise the framework. In this recursive process, we do not claim to see the knowledge of the particular teacher; instead, we use her teaching as a catalyst for investigating teacher knowledge, testing the usefulness of the framework, and providing feedback for improving the framework.

Figure 1 illustrates the flow of this process when three kinds of alternatives are explored: 1) the position and role of the lesson in the curriculum; 2) the major purposes of the lesson; 3) the teacher’s beliefs and commitments about mathematics teaching and learning. Clearly, other variables could be altered to generate different views of knowledge. For the lessons we studied, these three categories were useful in helping us generate ideas about teacher knowledge.
Figure 1: Video Viewing Flow Chart
Data

We used three sources for video data: Videocases for Mathematics Professional Development (VCPMD) cases (Seago, 2004; Seago et al., 2004); TIMSS public release video (Stigler et al., 1998); and commercial video from the Annenberg/CPB collection (Annenberg/CPB, 1996). From these collections, we viewed a total of nine videos as shown in Table 1.

Table 1: Videos used in the analysis

<table>
<thead>
<tr>
<th>VCPMD</th>
<th>Cindy Year 1 (Polygons)</th>
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<tbody>
<tr>
<td></td>
<td>Cindy Year 2 (Polygons)</td>
</tr>
<tr>
<td></td>
<td>Debbie (Pool border)</td>
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<tr>
<td></td>
<td>Kirk (Growing dots)</td>
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<tr>
<td>TIMSS</td>
<td>US 8th Grade Algebra</td>
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<tr>
<td></td>
<td>Japan 8th grade Coin Problem</td>
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<td></td>
<td>TIMSS-R Japan (inequalities)</td>
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<tr>
<td>Annenberg/CPB</td>
<td>Mr Cabana (Group test on sequences)</td>
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<td></td>
<td>Ms. Shimizu-Yost (Linear and exponential functions)</td>
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The materials available with the videos varied. The VCPMD videos included lesson graphs, transcripts, and pre- and post- interviews with the teachers. In addition, we were able to consult with the video case authors, Nanette Seago and Judy Mumme. The TIMSS videos were edited video, with transcripts but no accompanying materials. The Annenberg/CPB videos were edited and included clips of interviews with the teachers. We found the VCPMD videos most useful because of the extensive information available, and because they were unedited records of classrooms. The teacher interviews were invaluable in understanding some of the contextual issues for our initial analyses.
Analysis

Perhaps the most productive video we watched were three classes in the VCMPD series, in which two middle school teachers taught similar lessons involving geometric patterns as the site for doing algebra. In two of these videos, one teacher – Cindy -- taught the same lesson to different classes a year apart. In the third video, a different teacher – Debbie – taught a lesson similar in content.

*Cindy’s lesson*

Cindy’s lessons used a growing geometric pattern, beginning with a line of triangles and moving to squares, pentagons, and hexagons. In its general form, the problem is as follows:

![Diagram of a growing geometric pattern]

What is the perimeter of a chain of regular polygons where s is the length of the side, n is the number of polygons in the chain, and p is the number of sides in each polygon?

Cindy introduced the problem with the following question: “If we line up a hundred equilateral triangles in a row, what would be the perimeter?” The first year’s class raised many interesting issues about the mathematical knowledge entailed in teaching with this problem:
• Which of several possible variables should students attend to as variables? Possibilities include the number of polygons, the length of each side, the number of sides, and the total perimeter.

• What relationship(s) between variables should be used and in what forms or representations? This includes both the recursive explanation of the relationship between number of polygons and perimeter, and the functional explanation. Representations include tables, graphs, and equations (or expressions).

• For what purposes should this problem be used? Possibilities include exploring any of the variables or representations mentioned above and making connections among them. The problem can be used as an introduction to linear functions or proportional relationships. It could be used to reinforce understanding of perimeter or to explore linear functions.

For Cindy’s first class, it was the start of a unit on algebraic expressions, and she planned to get them thinking about different ways of representing patterns. We noted the following issues with respect to teacher knowledge:

a) Definitions: At the start of the class, Cindy takes time to tell students what various terms mean – perimeter, equilateral, regular, triangle.

b) Use of “unit”: There is much ambiguity in the class about what “unit” means. Unit of measure and length of a side are both referred to as unit. Cindy does not make the distinction between these two, or clarify her meaning to the students. This decision obscures one of the possible variables in the problem, the length of the side.
c) Connection between geometry and algebra: Cindy pushes to get equivalent expressions for the triangle pattern, irrespective of how the student visualized the relationship between the geometry and the algebraic expression. Every expression was “reduced” to a common form, the same as the others, without discussion of how the expression was seen in the geometry. Cindy seemed confused (and confirmed in her interview that she was puzzled) by one student’s interpretation of the perimeter as an algebraic expression that included the term “+4”. She did not have a geometric interpretation of what the student meant, and later suggested that getting 4 “as the intercept” was problematic. She did not try to help students see an expression as an object in its own right with geometric meaning.

In the end, Cindy’s lesson focused on generating equivalent expressions in one variable, simplifying each suggestion to the same standard form, n+2. In her interview, Cindy said that they had gained experience in working with patterns and algebraic expressions for those patterns.

In alternative versions of this lesson, a teacher might have chosen to exploit the relationship between the geometry and the algebra, emphasizing the differences in algebraic representations depending on one’s interpretation of the geometry, before reducing all expressions to an identical form. Or a teacher might emphasize the use of multiple variables, or the linearity of the resulting functions. Any of these versions would call on knowledge that we could not “see” in Cindy’s lesson.

In Cindy’s second video, she used the lesson at a different point in the curriculum to focus on linear functions rather than to introduce patterns as a precursor to linear functions. In this lesson, she was more explicit about getting expressions into the
standard form, and less surprised by alternative versions. She still did not focus on attaching meanings to equivalent expressions.

Debbie’s lesson

In Debbie’s lesson, students investigated the area of a tiled border around a square pool, starting with a 5x5 pool, and considering other sizes. She asked students to find an expression that worked for any size pool. Debbie had her students explore the differences in the expressions they generated, and asked them how they could have so many different answers for the same problem. Equivalence is an important concept in Debbie’s lesson, and she demonstrates a clear connection between geometry and algebra through the idea of equivalence. Debbie’s lesson also raises some questions about teacher knowledge:

a) Concepts. This problem could be used to focus on many different ideas, including area, perimeter, and their relationship; linear functions; expressions; and much more. What to emphasize, and how to relate different aspects of the problem, depends on the purpose of the lesson, and in important ways, on the teachers’ understanding of connections across these topics.

b) Equivalence. What does it mean to fully understand the concept of equivalence? How does equivalence relate to the “structural aspects of algebra” (Kieran, 1993)? Debbie seems to use equivalence as a leverage point for connecting algebra and geometry and for making the structure of expressions visible. What knowledge does she have that allows her to teach algebra this way?
Using the framework

Useful in “seeing” the knowledge these teachers bring to bear on their teaching are the “overarching categories” in our proposed framework for understanding knowledge for algebra teaching: Trimming, bridging, and decompressing.

Trimming

In both of these lessons, the teachers make decisions about how to present and use what could be fairly complicated problems at a fairly basic level. In Debbie’s case, she presents the problem at high level of abstraction, asking her students to come up with a general rule for any size pool. Then she starts them off with a 5x5 example and helps them work through it. Cindy similarly starts her students with a simple version of the more general problem, in her case without posing the generalized problem first.

What is different about these two teachers’ trimming, and what are the implications for teacher knowledge? In Cindy’s case, the way she presented the problem was appropriate, but she went on to lose some of the mathematical entailments of the problem through her handling of unit, and by ignoring the relationship between the geometry and the algebra. Her version of trimming did not retain some of the most powerful mathematics the problem represented. Did she know this mathematics? That is impossible to say, but it is fair to hypothesize that, before making such a move, a teacher should understand the mathematical implications.

Debbie made a similar initial trimming move, starting with a 5x5 pool, but she used the simpler example to more fully explore a range of mathematical ideas, retaining the integrity of the general problem as she did so.
Bridging

These classes both provide multiple opportunities for bridging: between algebra and geometry, across different representations of linear functions (table, expression, function, graph), and from less to more general abstraction. Within these categories are multiple big ideas, including equivalence, units of measure, recursive relationships, and more. One of the most striking differences between these two teachers was that Debbie made numerous links across the many representations and topics, while Cindy made few. Cindy seemed to be focused on getting students to come to a conclusion and create the correct (reduced) expression. Debbie was focused on helping students understand the complex mathematical relationships – geometric and algebraic – that the problem entailed. Debbie’s lesson was full of bridging, while Cindy’s was sparse.

Decompressing

Decompressing refers to unpacking mathematics in ways that expose meaning and aid understanding. We call it decompressing or unpacking (after Ball and Bass) to emphasize that it is reversing something that has been done previously. So, for example, in algebra, we take as given that “ab” means “a times b” when we write an expression like $a^2 + ab$. It has been noted that, early in the learning of algebra, students may mistake ab for the number with a in the tens position and b in the ones position. Decompressing this would mean explicitly calling attention to the meaning of the representation ab. Ball and Bass have provided multiple examples of unpacking in arithmetic (CITE).

In the lessons of Cindy and Debbie, perhaps the clearest example of decompressing is Debbie’s pursuit of the connection between the geometry of the pool and students’ algebraic representations of that geometry, especially in contrast to
Cindy’s rush to compress students’ solution into reduced form. Debbie makes explicit (and thus decompresses) the geometry behind the expressions (for the border of an nxn pool) $4n+4, 4(n+1), 2(n+2) + 2n, (n+2)^2 - n^2$. She helped students give geometric meaning to algebraic expressions, and provided a foundation for understanding what it means for two expressions to be equivalent.

As these examples make clear, the concepts of trimming, bridging, and decompressing are not crisply separated – an example of bridging can also illustrate trimming or decompressing, etc. They have proven to be useful categories for talking about what teachers are doing, and, for purposes of this research, for “seeing” knowledge in action.

Conclusions

Video has proven to be a very useful tool for us in exploring teacher knowledge. Our system of conjecturing, and creating alternative assumptions about the lesson, has given us a way to analyze and hypothesize about teacher knowledge without having to make specific claims about a particular teacher’s knowledge. Chomsky’s (1965) distinction between competency and performance has been useful, providing a way for thinking about the difference between what teachers were capable of doing (competence) and what they actually did on a given occasion (performance). Cindy might, for example, know the relationship between the geometry of the growing triangles problem and the various algebraic expressions implied, yet choose not to make this connection explicit. We have found this distinction important to keep in mind as we talk about particular teachers’ knowledge: a teacher may have competence (knowledge) in a particular area, but choose not to use it at a specific moment. We have no way of judging
whether the knowledge is there or not, if it is not used in practice. Thus, we generally make few claims about a specific teacher’s knowledge but rather focus on the knowledge that might be used in teaching particular mathematics as it unfolds in the classroom.

Our hypotheses about teacher knowledge have been aimed at creating a framework for developing items to assess knowledge (Ferrini-Mundy et al., 2005). We have gone back and forth between proposed elements of a framework and data from video and other sources. The framework, shown in Figure 2, has been through multiple versions. In its current form, it provides “sites” for identifying knowledge: tasks of teaching where knowledge is used, aspects of mathematics that call on different forms of knowledge, and the “overarching categories” that seem to be windows into the depth and breadth knowledge that might be used in algebra teaching.
Figure 2: Framework for analyzing knowledge for algebra teaching

Acknowledgements

This research is supported by National Science Foundation award REC 0106709 and REC-0337595. Contributing to the empirical work on the first project were Betsy Becker, Daniel Chazan, Robin Marcus, Kelly Hodges, and Jack Smith as well as the author. PI’s for the second project, Knowledge for Algebra for Teaching (KAT), are Joan Ferrini-Mundy, Robert Floden, Sharon Senk, and the author. The author thanks Judy Mumme and Nanette Seago and the VCPMD project for videos of classroom teaching, and for their helpful feedback on our work.
Comments, questions, and feedback should be directed to the author, mccrory@msu.edu.

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