**Introduction:**

This interview will focus on algebra and algebra teaching.

Let’s start off by having you tell me a little bit about yourself as a teacher of algebra:

- When did you start teaching algebra? Where? How long have you taught algebra?
- What kinds of curricula have you used? (prepare some specific prompts? what are we interested in here? Computers, manipulatives, specific curricula, textbook series?)  We may have to prompt here to ask whether they’ve had experience with any “innovative” algebra curricula…
- Do you enjoy teaching algebra? (Prompts: Why? In what ways? Why not?)
- Does it have particular challenges?
- Have there been any important influences on how you teach algebra? Tell me about that… (probe whether there was something specific to algebra)

**Part One: Equations overview**

How might you explain what an equation is to a student?

Is there anything that you might not tell your student, but that is an important aspect of an equation to you?

To make our conversation more specific, I would like to start by having you look at some index cards. Please look these index cards over for a minute and think about how you teach students about equations and what they are.

A. \( y = 3x - 4 \)
B. \( f(x) = 3x - 4 \)
C. \( 9x - 3y = 12 \)
D. \( 3x - 4 = 12 \)
E. \( 3(x - 2) + 2 = 4x - 4(x - 3) \)
F. \( 4(x - 1) - x = 3x - 4 \)

To get started, and understand how you think about communicating what an equation is to students, would you tell students that each of these is an equation or are there any that you would exclude? If so, why? If not, why not? What is it that makes something an equation?

[Here we hope to see whether they define an equation as a string with an equal sign, or whether it is a comparison of two functions, or a statement that can be true or false. We will use this information with other information that comes up later in the interview.]

Do you see the equations on these cards all as basically alike, or are some different? If some feel different tell me what makes some seem different to
you and others alike. For example, you could group the cards into groups that seem similar to each other and different from others. [We’d like to see what sorts of views of the equal sign and letters show up in how they talk about this. Also, do they use linear and non-linear, or numbers of variables to talk about similarities and differences between equations.]

A key problem in teaching is figuring out how to help students learn about complicated things like equations. What sorts of equations should students meet first? What ones should they meet second? And, so on. Here are three more cards.

G. \( g(x,y) = 3x-y \)
H. \( 3x^2+x-2 = 0 \)
I. \( y = 3x^2+x-2 \)

Using this set of nine cards, if you could choose your own ideal ordering, in what order would students encounter these cards in your introductory algebra course?

[The interviewer will look for one of three classic orderings: equations in 1 variable, followed by equations in 2; linear equations followed by non-linear; functions first, comparisons of functions second.]

<table>
<thead>
<tr>
<th>Functions, then comparisons of functions</th>
<th>Equations of one variable, then equations of two variables</th>
<th>Linear equations first, then non-linear equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=3x-4</td>
<td>( 3x - 4 = 12 )</td>
<td>( 3x - 4 = 12 )</td>
</tr>
<tr>
<td>Y=3x^2+x-2</td>
<td>( 3(x-2)+2=4x-4(x-3) )</td>
<td>( 3(x-2)+2=4x-4(x-3) )</td>
</tr>
<tr>
<td>f(x) = 3x - 4</td>
<td>( 4(x-1)-x=3x-4 )</td>
<td>( 4(x-1)-x=3x-4 )</td>
</tr>
<tr>
<td>G(x,y) = 3x-y</td>
<td>( 3x^2+x-2=0 )</td>
<td>( Y=3x-4 )</td>
</tr>
<tr>
<td>3x-4=12</td>
<td>( Y=3x-4 )</td>
<td>( 9x-3y=12 )</td>
</tr>
<tr>
<td>3x^2+x-2=0</td>
<td>( 9x-3y=12 )</td>
<td>( f(x) = 3x - 4 )</td>
</tr>
<tr>
<td>3(x-2)+2=4x-4(x-3)</td>
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<td>( 3x^2+x-2=0 )</td>
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<td>( G(x,y) = 3x-y )</td>
<td>( G(x,y) = 3x-y )</td>
</tr>
</tbody>
</table>

If the ordering is not exactly one of these, the INTERVIEWER WILL RECORD THE ORDERING BY NUMBERING A LIST—use N/A for examples they would not use]

Why do you think this order is the best order to use? (prompts: Was this order how you learned yourself?)

Use one of the three canonical orderings and ask: Other people we have spoken with prefer this order. What would you think about this order?
Which of these cards is the best example to use to explain to your students what an equation is? or is there something else you prefer to use?

Which of these cards is the best example to use to explain to your students what it means to “solve an equation” and what is a “solution of an equation”? Is there something else you prefer to use?)

Here is an identical set of nine cards. Order this set in the order in which you think students encounter each type of example in your school’s curriculum?

[INTERVIEWER WILL RECORD THE ORDERING BY NUMBERING A LIST with n/a for those note used.]

(Prompt: If there isn’t a match between this ordering and the ideal ordering, how do you cope with this difference? How did you get to your point of view?)

[Potential follow ups:
  • Which of these examples is the best to use to explain to your students what an equation is or is there something else you prefer to use?
  • Which of these examples is the best to use to explain to your students what it means to “solve an equation” and what is a “solution of an equation”? Is there something else you prefer to use?]

We will analyze the orderings in terms of: uses of letters, uses of the equals sign, the number of variables. Also look for what distinctions the teacher makes.

Now, let’s make our conversation even more specific and look at a number of these examples together and a problem that could be asked of each one: [These 3 equations will be on index cards in the partial set.]

- \( y = 3x - 4 \)
- \( f(x) = 3x - 4 \)
- \( 3x - 4 = 12 \)

Consider this problem. [This problem will be on a strip of paper.]

If \( f(x) = 3x - 4 \), what value of \( x \) makes \( f(x) = 12 \)?

How does this problem compare to the two following problems? [These two problems will be on strips of paper.]

For the expression \( 3x - 4 \), what value of \( x \) makes that expression equal to 12?
If \( y = 3x - 4 \), what value of \( x \) makes \( y = 12 \)?

[Ask for a comparison to two other ways of asking the same question. If similar, ask if they are exactly the same or specific differences exist between them.]

Now let’s look at some solutions to this sort of problem. Here is one student’s solution to the original problem: [This solution will be on a separate page.]

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**Sam’s solution**

“OK, this means that 12 has to equal 3\( x \) - 4, so I can write:” \( 12 = 3x - 4 \)

“Now I can solve this by adding 4 to both sides.” \( 16 = 3x \)

“Now I’ll divide by 3.” \( 16/3 = x \)

“16 divided by 3 is 5.33333…, which I can round to 5.3.” \( 5.3 = x \)

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Does Sam’s work seem like an acceptable solution to the original problem? How come? Does it seem like an acceptable solution to either of the two alternative problems. [Ask for connections between the Sam’s solution and the problem versions.] Does it seem like an unacceptable solution to any of the problems?

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Here is another student’s solution: [This solution will be on a separate page.]

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**Karim’s solution**

“I can solve this with a table. I’ll start with \( x = 4 \).” He draws the following table:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“\( x = 4 \) gets me 8, \( x = 5 \) gets me 11, and \( x = 6 \) gets me 14.”

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>

“So \( x \) has to be between 5 and 6.”

“12 is closer to 11 than 14, so \( x \) has to be closer to 5 than 6, so \( x \) is about 5.3.”
What distinctions does the teacher make between the different forms of the problem? What differences does the teacher perceive in the use of the equal sign and in the use of letters in these problem contexts?

Now let’s look at the examples: [These equations will be on index cards in the partial set.]

- \( y = 3x - 4 \)
- \( 9x - 3y = 12 \)
- \( 3x - 4 = 12 \)
- \( 2(x + 1) = 4x - 3 - x \)
- \( 4(x - 1) - x = 3x - 4 \)

How do you interpret the equal sign in the first two? How about in the third one? In the fourth and fifth ones?

Could the equal sign in any of these examples be interpreted differently? [If yes, explain how and:

- Do you think that it’s important to help students learn to interpret the equal sign in different ways?
- How do you (or would you) do that? ]

What distinctions does the teacher make between the various uses of the equal sign? How important does the teacher believe these differences to be? How does the teacher address these differences with algebra students?

We looked at Sam’s and Karim’s solutions of the equation \( 3x - 4 = 12 \). How is solving for \( x \) in the equation \( 9x - 3y = 12 \) similar to and different from solving for \( x \) in the equation \( 3x - 4 = 12 \)?

Describe the solution set of \( 9x - 3y = 12 \).

[Possible follow-up questions:

- How is this equation similar to and different from \( 3x - 4 = 12 \) or \( 2(x + 1) = 4x - 3 - x \)?
- What does it mean for you to solve this equation?
- How would you explain to your students what it means to solve this kind of equation as opposed to other kinds?

]
Suppose you ask your students to solve $4(x-1) - x = 3x - 4$.

What questions or difficulties might you expect students to have?

How might you respond if a student asked you to explain why you can get $0=0$ when solving an equation?

Some students interpret $0=0$ as $x=0$. Why do you think students do this? How do you, or how would you, address this issue in your algebra class?

[If needed] How would you help your students to interpret this solution?

[If the teacher does not suggest a graphical approach, an example of such an approach will be on a separate page]:

One thing some teachers do is to graph both sides of the equation. Students can compare tables of values and graphs of both sides. What do you think about this approach to solving this problem or explaining the solution to students?

Show me how you would solve the following two equations simultaneously:

- $y = 3x - 4$
- $9x - 3y = 12$

How do you think about the solution to this problem? How would you explain the solution of this type of problem to your students?

[If the teacher does not suggest a graphical approach:] Would you think about a graphical approach for solving this problem or explaining it? Compare the graph of this system to the graph used to solve the previous equation. How are they alike? Are they different in any way?

How does the first equation $[4(x-1) - x = 3x - 4]$ compare to the following problem: [This system will be on an index card in the partial set.]

Solve $\begin{cases} y = 4(x-1) - x \\ y = 3x - 4 \end{cases}$

Is this the same problem? How is it similar to and different from the single equation? Where would this problem fit into your orderings?

*Does the teacher primarily focus on letters as unknowns or as variables? How does the teacher view the equals sign in the single equation? Does the teacher distinguish between equations in one
variable with the variable on both sides and systems of equations in two variables? Does the teacher perceive differences in the uses of graphs in these two contexts?

[If the teacher’s responses to the preceding questions suggest a view of letters as primarily unknowns]:

Now let’s look at the examples: [These will be on index cards.]

• \(\sqrt{(3x - 4)^2} = 3x - 4\)
• \(2^x = x^2\)

How are these equations similar to and different from previous equations we have discussed?
How would you solve these equations?
Why are these equations difficult?
Would you consider a graphical approach to solve either of these equations?
Do you consider these good algebra problems? Would you ask your algebra students to solve equations like these? If yes, how would you approach the solution of these equations with your students?

**Part Two: Expressions (+ Inequalities)**

Equations are the cornerstone of school algebra, but of course, equations are not the only mathematical objects we study in algebra. At this point, I would like to turn our discussion to expressions and inequalities.

Suppose you surveyed a new Algebra class to see what their understanding of expressions and equations were from their pre-algebra experiences. Students said the following things: [These statements will be on a separate page]

a. An equation is something you work out to get a number.
b. An expression is an answer.
c. An expression is a number.
d. An expression is just a procedure.
e. An equation is an exercise.

• What do you think of the students’ responses.
• How would you use this information?

How does the teacher distinguish between equations and expressions? How does the teacher address these distinctions with algebra students?
Consider the following tasks: [These will be on index cards.]

- Solve for $x$: $2(x + 1) = 4x - 3 - x$.
- Evaluate $2(x + 1)$ for $x = 5$.
- Simplify $4x - 3 - x$.
- Solve for $x$: $2(x + 1) \leq 4x - 3 - x$.

In relation to your previous orderings, both the order of the curriculum and your preferred order, where would the last three tasks fit?

[Ask about the teacher’s perceived relative difficulty of the tasks, connections between the tasks, and whether any of these tasks might be presented together.]

*Are there differences in the teacher’s perceived uses of letters when the context changes from equations to expressions or inequalities?

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Suppose you ask a class to simplify $4x - 3 - x$.

Several students do the following:

\[
4x - 3 - x = 0 \\
3x - 3 = 0 \\
3x = 3 \\
x = 1
\]

Why do you think this happens? As a teacher, how would you proceed?

*Does the teacher talk about different uses of letters?*

I have one more thing I’d like to discuss with you, but before we move to that, do you have any questions for me?

As you know, we would like to visit your classroom to see how you address some of the issues we have discussed in your algebra teaching. I want to share a lesson with you that relates to what we’ve been talking about.

Core-Plus lesson (Course 1, Unit 3, Lesson 3, Investigation 1)—where does it fit? What would students need to know? What sorts of issues does it address?

Would you be willing to use this lesson with your students at some time and have me visit?